Belief Change

From AGM to Realistic Models

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Course Outline

- **Lecture 1:** AGM Theory
- **Lecture 2:** Base Dynamics
- **Lecture 3:** New Trends in Revision
- **Lecture 4:** The theory of update
- **Lecture 5:** Computational models of update

Belief Change — An example

(Gärdenfors & Rott 1995)

- Beliefs

  The bird caught in the trap is a swan
  The bird caught in the trap comes from Sweden
  Sweden is part of Europe
  All European swans are white

- Consequences

  The bird caught in the trap is white

- New information

  The bird caught in the trap is black

Which sentence(s) would you give up?

Motivation

Problem arises in several areas:

Databases: New entry inconsistent with database.

Robotics: Sensor information inconsistent with plans.

Diagnosis: Device behavior inconsistent with device description.

...

- Problem is not trivial:
  - Choice involved.
  - Indirect consequences of revision.
  - Representation issues.

Desiderata

We are looking for belief change operations that

- follow certain principles of rationality:
  - minimal change.
  - categorical matching.
  - weak consistency.
- tolerate some inconsistency.
- are computationally tractable.

Revision Versus Update

(Katsuno & Mendelzon)

- Revision: agent gets new information about static world.
  - Agent believes the book is either on the table or on the chair.
  - Agent is told the book is not on the table.
  - Agent believes the book is on the chair.
- Update: agent gets information about changes in the world.
  - Agent believes the book is either on the table or on the chair.
  - Agent is told that all objects that were on the table were removed from the room.
  - Agent believes that either the book is on the chair or it was removed from the room.

Lecture 1

AGM Theory

AGM Theory

Alchourrón, **G**ärdenfors, and **M**akinson (1985)

- Epistemic states: sets of formulas K.
- Epistemic attitudes:
  - $\alpha \in K$ - α accepted
  - $\neg\alpha \in K$ - α rejected
  - otherwise - α undetermined
- Input: formula
- Change operations: expansion, contraction, and revision.

Preliminaries

- Belief sets $K = Cn(K)$.
- Three operations:
  - Expansion ($K + \alpha = Cn(K \cup \{\alpha\})$)
  - Contraction ($K \dot{-} \alpha$)
  - Revision ($K * \alpha$)
- $K * \alpha = (K \dot{-} \neg\alpha) + \alpha$ (Levi identity)

For contraction and revision, rationality postulates.

Contraction Postulates

(**K-1**) $K \dot{-} \alpha$ is a belief set (*closure*)

(**K-2**) $K \dot{-} \alpha \subseteq K$ (*inclusion*)

(**K-3**) If $\alpha \notin K$, then $K \dot{-} \alpha = K$ (*vacuity*)

(**K-4**) If not $\vdash \alpha$, then $\alpha \notin K \dot{-} \alpha$ (*success*)

(**K-5**) If $\alpha \in K$, then $K \subseteq (K \dot{-} \alpha) + \alpha$ (*recovery*)

(**K-6**) If $\vdash \alpha \leftrightarrow \beta$, then $K \dot{-} \alpha = K \dot{-} \beta$ (*equivalence*)

Constructions for Contraction

- Partial Meet (AM)
- Safe Contraction (AM)
- Systems of Spheres (Grove)
- Epistemic Entrenchment (MG)

Partial Meet Contraction – 1

- Let K be a belief set and α a formula. For any set X, $X \in K \perp \alpha$ if and only if:
  - $X \subseteq Y$.
  - $X \nvdash \alpha$.
  - For all X' such that $X \subset X' \subseteq K$, $X' \vdash \alpha$.
- A selection function g must satisfy:
  - $g(K \perp \alpha) \subseteq K \perp \alpha$.
  - If $K \perp \alpha \neq \emptyset$, then $g(K \perp \alpha) \neq \emptyset$
    otherwise, $g(K \perp \alpha) = \{K\}$

Partial Meet Contraction – 2

Example: Let $K = Cn(p \wedge q)$

- $K \perp p = \{Cn(p \leftrightarrow q), Cn(q)\}$
- $K \perp p \wedge q = \{Cn(p), Cn(q), Cn(p \leftrightarrow q)\}$
- $K \perp p \rightarrow q = \{Cn(p), Cn(q)\}$

We may have:

- $g(K \perp p) = \{Cn(p \leftrightarrow q)\}$
- $g(K \perp p \wedge q) = \{Cn(p), Cn(q)\}$
- $g(K \perp p \rightarrow q) = \{Cn(p), Cn(q)\}$

Partial Meet Contraction – 3

Definition: Let K be a belief set, α a formula, and g a selection function. The partial meet contraction of K by α is given by:

$$K \dot{-} \alpha = \bigcap g(K \perp \alpha).$$

Theorem (AGM): An operation $\dot{-}$ on K is a partial meet contraction if and only if it satisfies postulates (**K-1**)-(**K-6**).

Limit Cases – Full meet

- All elements of $K \perp \alpha$ are selected, that is:

  $$K \dot{-} \alpha = \bigcap (K \perp \alpha).$$

  No choice involved.
- Problem: revision can lead to a set which is too small.

  If a revision operation is defined from full meet contraction by means of the Levi identity, then, for any α such that $\neg\alpha \in K$, $K * \alpha = Cn(\alpha)$.

Limit Cases – Maxichoice

- Only one element of $K \perp \alpha$ is selected, that is:

  $$K \dot{-} \alpha = \bigcap g(K \perp \alpha) \in K \perp \alpha.$$

- Problem: revision may lead to a set which is too big.

  If a revision operation is defined from a maxichoice contraction by means of the Levi identity, then, for any α such that $\neg\alpha \in K$, $K * \alpha$ will be maximal, i.e., for every formula β, either $\beta \in K * \alpha$ or $\neg\beta \in K * \alpha$.

Limit Cases – Examples

- Full meet: Suppose I believe that p (Buenos Aires is the capital of Brazil) and that q (there is no King of France). When I learn $\neg p$ and revise my belief set using a revision operation based on full meet contraction, I give up the belief that there is no King of France.
- Maxichoice: Suppose I believe p (that Buenos Aires is the capital of Brazil) and have no idea about q (that the King of France is bald). Finding out that $\neg p$ is the case and revising my belief set using a revision based on maxichoice contraction means that I will make a decision as to q or $\neg q$.

Safe Contraction – 1

- Let K be a belief set and α a formula. For any set X, $X \in K \perp\!\!\!\perp \alpha$ if and only if:
  - $X \subseteq Y$.
  - $X \vdash \alpha$.
  - For all X' such that $X' \subset X \subseteq K$, $X' \not\vdash \alpha$.
- Let $<$ be an acyclical hierarchy over K. An incision function σ selects from each element of $K \perp\!\!\!\perp \alpha$ the minimal elment w.r.t. $<$:
  - $\sigma(K \perp\!\!\!\perp \alpha) \subseteq \bigcup(K \perp\!\!\!\perp \alpha)$.
  - If $X \in K \perp\!\!\!\perp \alpha$, $X \neq \emptyset$, β minimal in X, then $\beta \in X \cap \sigma(K \perp \alpha)$.

Safe Contraction – 2

Definition: Let K be a belief set, α a formula, and σ an incision function. The safe contraction of K by α is given by:

$$K \dot{-} \alpha = K \setminus \sigma(K \perp\!\!\!\perp \alpha).$$

Theorem (AM85): An operation $\dot{-}$ on K is a safe contraction if and only if it satisfies postulates (**K-1**)-(**K-6**).

Systems of Spheres – 1

Possible worlds semantics:

- $\mathcal{W}$ - set of all possible worlds (or truth assignments).
- $[\alpha] = \{w \in \mathcal{W} | w(\alpha) = 1\}$
- $[A] = \bigcap\{[\alpha] | \alpha \in A\}$
- $t(w) = \{\alpha | w(\alpha) = 1\}$
- $t(W) = \{\alpha | \forall w \in W, w(\alpha) = 1\}$ (Theory!)

System of Spheres – 2

Let S be a family of sets of possible worlds such that:

(S1) S is totally ordered by $\subseteq$.
(If $U, V \in S$, then $U \subseteq V$ or $V \subseteq U$)

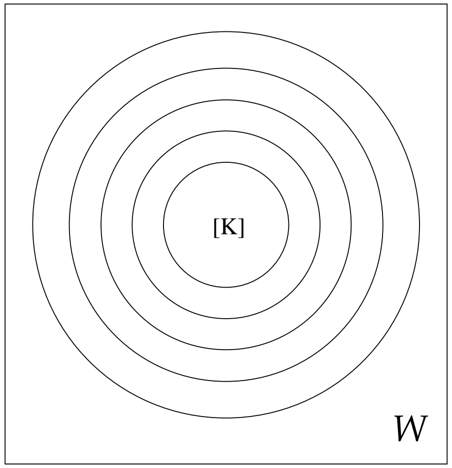
(S2) $[K]$ is the minimal element.
($[K] \in S$ and $\forall U \in S, [K] \subseteq U$)

(S3) $\mathcal{W}$ is the maximal element.

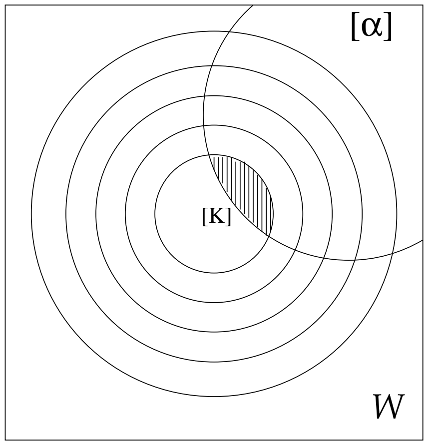
(S4) If any sphere intersects $[\alpha]$, then there is a sphere $c(\alpha)$ that is the smallest sphere intersecting $[\alpha]$).

Let $f(\alpha)$ denote $c(\alpha) \cap [\alpha]$.

Systems of Spheres – 3

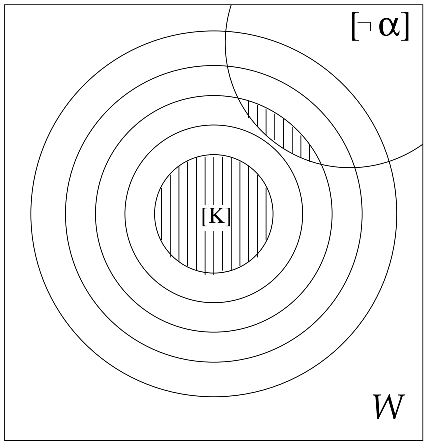


Systems of Spheres – Expansion



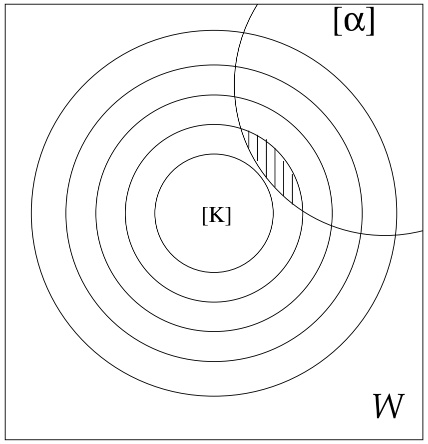
$$K + \alpha = t([K] \cap [\alpha])$$

Systems of Spheres – Contraction



$$K \dot{-} \alpha = t([K] \cup f(\neg\alpha))$$

Systems of Spheres – Revision



$$K * \alpha = t(f(\alpha))$$

Systems of Spheres – Results

Definition: Let K be a belief set and α a formula. The sphere contraction of K by α is given by:

$$K \dot{-} \alpha = t([K] \cup f(\neg\alpha))$$

Theorem (Grove 88): An operator $\dot{-}$ is a sphere contraction if and only if it satisfies postulates (**K-1**)-(**K-6**) and
(**K-7**) $K \dot{-} \alpha \cap K \dot{-} \beta \subseteq K \dot{-} (\alpha \wedge \beta)$
(**K-8**) If $\alpha \notin K \dot{-} (\alpha \wedge \beta)$, then $K \dot{-} (\alpha \wedge \beta) \subseteq K \dot{-} \alpha$

Epistemic Entrenchment – 1

- (Transitivity) If $\alpha \leq_K \beta$ and $\beta \leq_K \gamma$, then $\alpha \leq_K \gamma$.
- (Dominance) If $\alpha \vdash \beta$, then $\alpha \leq_K \beta$.
- (Conjunctiveness) $\alpha \leq_K (\alpha \wedge \beta)$ or $\beta \leq_K (\alpha \wedge \beta)$.
- (Minimality) If $K \neq \perp$, then $\alpha \notin K$ if and only if $\alpha \leq_K \beta$ for all β.
- (Maximality) If $\beta \leq_K \alpha$ for all β, then $\vdash \alpha$.

Epistemic Entrenchment – 2

- (**C**$\leq$) $\alpha \leq_K \beta$ if and only if $\alpha \notin K \dot{-} (\alpha \wedge \beta)$ or $\vdash (\alpha \wedge \beta)$.
- (**C**$\dot{-}$) $\beta \in K \dot{-} \alpha$ if and only if $\beta \in K$ and either $\vdash \alpha$ or $\alpha \leq_K (\alpha \vee \beta)$.

Theorem (GM88): If an ordering $\leq$ satisfies **(EE1)-(EE5)**, then the contraction function determined by (**C**$\dot{-}$) satisfies **(K-1)-(K-8)** as well as condition (**C**$\leq$).

Theorem (GM88): If a contraction operator $\dot{-}$ satisfies **(K-1)-(K-8)**, then the ordering $\leq$ that is determined by (**C**$\leq$) satisfies **(EE1)-(EE5)** as well as condition (**C**$\dot{-}$).

Recovery: Example 1

(Hansson)

I believe that "Cleopatra had a son" (α) and that "Cleopatra had a daughter" (β), and thus also that "Cleopatra had a child" ($\alpha \vee \beta$, briefly δ). Then I receive information that makes me give up my belief in δ, and contract my belief set accordingly, forming $K \dot{-} \delta$. Soon afterwards I learn from a reliable source that "Cleopatra had a child". It seems perfectly reasonable for me to then add δ (i.e. $\alpha \vee \beta$) to my set of beliefs without also reintroducing either α or β.

Recovery: Example 2

I previously entertained the two beliefs, "x is divisible by 2" (α) and "x is divisible by 6" (β). When I received new information that induced me to give up the first of these beliefs (α), the second (β) had to go as well (since α would otherwise follow from β).
Then I received new information that made me accept the belief "x is divisible by 8". (ε). Since α follows from ε, $(K \dot{-} \alpha) + \alpha$ is a subset of $(K \dot{-} \alpha) + \varepsilon$, then by recovery I obtain that "x is divisible by 24" (δ), contrary to the intuition.

Success: Examples

(Fermé)

"Yesterday, the Pope called me to wish me good luck in the tutorial."

"Yesterday, my mother called me to wish me good luck in the tutorial."

One day when you return back from work, your son tells you, as soon as you see him: "A dinosaur has broken our Mings dynasty vase in the living-room". You probably accept one part of the information, namely that the vase has been broken, while rejecting the part of it that refers to a dinosaur.

Give up your belief in P $\neq$ NP, since you do not have a proof for it.

Lecture 2

Base Dynamics

Bases

Problems with the use of logically closed belief sets:

- Infinite sets.
- Inconsistency leads to trivialization.
- Highest priority to incoming information.
- No distinction between explicit and implicit beliefs.

Reasons for Using Belief Sets

- Syntax independence
  - what matters is the content, not the form.
- Knowledge level in AI
  - coexists with other levels of description.
- Logical elegance.

Belief Bases (à la Hansson)

- Belief base B finite set of formulas.
- Expansion: $B + \alpha = B \cup \{\alpha\}$.
- Epistemic attitudes:
  - $\alpha \in Cn(B)$: α (implicitly) believed.
  - $\alpha \in B$: α explicitly believed.
  - $\alpha \in Cn(B) \setminus B$: α merely derived.

Example (Hansson)

- α: Paris is the capital of France.
- β: There is milk in the fridge.
- $\alpha, \beta \in B \Rightarrow \alpha \leftrightarrow \beta \in Cn(B)$

  When we revise by $\neg\beta$, we must choose between giving up α and $\alpha \leftrightarrow \beta$.

  In the belief base approach, $\alpha \leftrightarrow \beta$ is automatically chosen and α remains in the revised base (**"Disbelief Propagation"**).

More advantages of the use of bases

Expressivity $B_1 = \{\alpha, \beta\}$, $B_2 = \{\alpha, \alpha \leftrightarrow \beta\}$.

$Cn(B_1) = Cn(B_2)$

$B_1 * \neg\alpha = \{\neg\alpha, \beta\}$

$B_2 * \neg\alpha = \{\neg\alpha, \alpha \leftrightarrow \beta\}$

$\beta \in Cn(B_1 * \neg\alpha)$, but $\beta \notin Cn(B_2 * \neg\alpha)$.

Inconsistency Tolerance $B_1 = \{p. \neg p, q_1, q_2, q_3\}$

$B_2 = \{p, \neg p, \neg q_1, \neg q_2, \neg q_3\}$

$Cn(B_1) = Cn(B_2)$, but $Cn(B_1 \dot{-} p) \neq Cn(B_2 \dot{-} p)$

Operations on Belief Bases

- **Expansion:** $B + \alpha = B \cup \alpha$
- **Contraction:**
  - Partial meet.
  - Kernel.
  - Prioritized.
- **Revision:**
  - Internal.
  - External.
- **New operations.**

Partial Meet Base Contraction Construction

- $B \perp \alpha$: maximal subsets of B that fail to imply α
- γ: function that selects some elements of $B \perp \alpha$
- $B \dot{-}_\gamma \alpha = \bigcap \gamma(B \perp \alpha)$

Partial Meet Base Contraction Postulates

- If $\alpha \notin Cn(\emptyset)$, then $\alpha \notin Cn(B \dot{-} \alpha)$ (success)
- $B \dot{-} \alpha \subseteq B$ (inclusion)
- If $\beta \in B \setminus (B \dot{-} \alpha)$, then there is some B' such that $B \dot{-} \alpha \subseteq B' \subseteq B$, $\alpha \notin Cn(B')$ and $\alpha \in Cn(B' \cup \{\beta\})$ (relevance)
- If for all subsets B' of B, $\alpha \in Cn(B')$ if and only if $\beta \in Cn(B')$, then $B \dot{-} \alpha = B \dot{-} \beta$ (uniformity)

Partial Meet Base Contraction Results

- Representation Theorem.
- Postulates not as intuitive as AGM.
- Recovery does not hold: $B = \{p \wedge q\}$, $(B \dot{-} p) + p = \{p\}$.
- Operation is really different from AGM. For example, maxichoice is not a problem for belief bases. It is actually often used in practice.

Kernel Contraction – Idea

- Generalization of safe contraction.
- No hierarchy of formulas.
- Selects at least one formula from every minimal set implying α.
- Unlike AGM partial meet/safe contraction, kernel contraction is more general than partial meet.

Kernel Base Contraction – Construction

- $B \perp\!\!\!\perp \alpha$: minimal subsets of B that imply α
- σ: function that selects at list one element of each set in $B \perp\!\!\!\perp \alpha$
- $B \dot{-}_{\sigma} \alpha = B \setminus \sigma(B \perp\!\!\!\perp \alpha)$

Kernel Base Contraction – Postulates

- If $\alpha \notin Cn(\emptyset)$, then $\alpha \notin Cn(B \dot{-} \alpha)$ (success)
- $B \dot{-} \alpha \subseteq B$ (inclusion)
- If $\beta \in B \setminus B \dot{-} \alpha$, then there is some $B' \subseteq B$ such that $\alpha \notin Cn(B')$ and $\alpha \in Cn(B' \cup \{\beta\})$ (core-retainment)
- If for all subsets B' of B, $\alpha \in Cn(B')$ if and only if $\beta \in Cn(B')$, then $B \dot{-} \alpha = B \dot{-} \beta$ (uniformity)

Contraction - Example

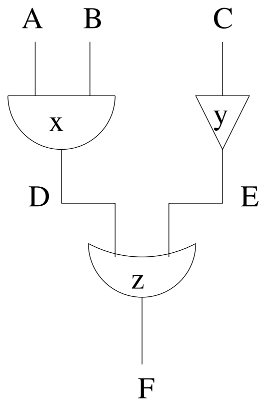
$B = \{p, p \vee q, p \leftrightarrow q\}$

$B \perp\!\!\!\perp (p \wedge q) = \{\{p, p \leftrightarrow q\}, \{p \vee q, p \leftrightarrow q\}\}$

$B \perp (p \wedge q) = \{\{p, p \vee q\}, \{p \leftrightarrow q\}\}$

$B \dot{-} (p \wedge q) = \{p\}$ can be constructed as kernel but not partial meet contraction.

Application – Diagnosis



Application – Diagnosis

$$\begin{aligned} B = \ & \{okX, okY, okZ, \\ & (A \wedge B) \wedge okX \rightarrow D, \neg(A \wedge B) \wedge okX \rightarrow \neg D, \\ & C \wedge okY \rightarrow \neg E, \neg C \wedge okY \rightarrow E, \\ & (D \vee E) \wedge okZ \rightarrow F, \neg(D \vee E) \wedge okZ \rightarrow \neg F\} \end{aligned}$$

OBS = $\neg C \wedge \neg F$.

$B \perp\!\!\!\perp \neg$OBS = $\{\{\neg C \wedge okY \rightarrow E, (D \vee E) \wedge okZ \rightarrow F, okY, okZ\}\}$

We have two possibilities for minimal incision functions:
$\sigma_1 = \{okY\}$ and $\sigma_2 = \{okZ\}$.

This means that either Y or Z are not working well.

Prioritized Bases – 1

Linear order (for example, temporal order):

- $B = (\alpha_1, \alpha_2, \ldots, \alpha_n)$
- $\alpha_i < \alpha_j$ for $i < j$
- $B_0 = \emptyset$

  $$B_{i+1} = \begin{cases} B_i \cup \{\alpha_{i+1}\} & \text{if } B_i \cup \{\alpha_i\} \not\models \alpha \\ B_i & \text{otherwise} \end{cases}$$

  $B \dot{-} \alpha = B_n$

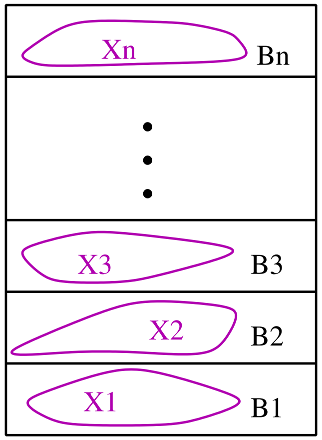
OBS.: $(B \dot{-} \alpha) \in B \perp \alpha$, i.e., contraction is maxichoice.

Problem: Assumption that order is linear.

Prioritized Bases – 2

- Total pre-order $\leq$:
  - reflexive, transitive, and for all α, β, either $\alpha \leq \beta$ or $\beta \leq \alpha$.
- B is divided in equivalence classes w.r.t. $\leq$.
- From each class, we pick a maximal consistent subset not implying α.
- Result of contraction is the union of the subsets of each class.

Prioritized Bases – Construction



Prioritized Bases – Construction 2

(Nebel)

$X \in B \Downarrow \alpha$ iff:

- $X = \bigcup X_i$
- $X_i \subseteq B_i$
- X_i is a maximal subset of B_i such that $\bigcup_{j \leq i} X_j \not\models \alpha$

$B \dot{-} \alpha = \bigcap (B \Downarrow \alpha)$

OBS.: $B \Downarrow \alpha \subseteq B \perp \alpha$, i.e., contraction is partial meet.

Revision of Belief Bases

- Levi Identity
  Internal revision:
  $B \mp \alpha = (B \dot{-} \neg\alpha) + \alpha$
- Reversed Levi Identity (Hansson)
  External revision:
  $B \pm \alpha = (B + \alpha) \dot{-} \neg\alpha$
  - Intermediate state may be inconsistent.

Revision of Belief Bases – Examples

Let $B = \{p \rightarrow r, p \rightarrow \neg r\}$.

(a) internal revision: $B \perp \neg p = \{\{p \rightarrow r\}, \{p \rightarrow \neg r\}\}$, hence,
for $\gamma_1(B \perp \neg p) = \{\{p \rightarrow r\}\}$, we have $B \mp_{\gamma_1} p = \{p \rightarrow r, p\}$.

(b) external revision: $B + p \perp \neg p = \{\{p \rightarrow r, p\}, \{p \rightarrow \neg r, p\}\}$, hence,
for $\gamma_2(B + p \perp \neg p) = \{\{p \rightarrow \neg r, p\}\}$, we have $B \pm_{\gamma_2} p = \{p \rightarrow \neg r, p\}$.

Internal Partial Meet Revision

(B*1) If $\neg\alpha \notin Cn(\emptyset)$, then $\neg\alpha \notin Cn(B \mp \alpha)$ (non-contradiction)

(B*2) $B \mp \alpha \subseteq B \cup \{\alpha\}$ (inclusion)

(B*3) If $\beta \in B \setminus B \mp \alpha$, then there is some B' such that $B \mp \alpha \subseteq B' \subseteq B \cup \{\alpha\}$, $\neg\alpha \notin Cn(B')$ but $\neg\alpha \in Cn(B \cup \{\beta\})$ (relevance)

(B*4) $\alpha \in B \mp \alpha$ (success)

(B*5) If for all $B' \subseteq B$, $\neg\alpha \in Cn(B')$ if and only if $\neg\beta \in Cn(B')$, then $B \cap (B \mp \alpha) = B \cap (B \mp \beta)$ (uniformity)

External Partial Meet Revision

(B*1) If $\neg\alpha \notin Cn(\emptyset)$, then $\neg\alpha \notin Cn(B \mp \alpha)$ (non-contradiction)

(B*2) $B \mp \alpha \subseteq B \cup \{\alpha\}$ (inclusion)

(B*3) If $\beta \in B \setminus B \mp \alpha$, then there is some B' such that $B \mp \alpha \subseteq B' \subseteq B \cup \{\alpha\}$, $\neg\alpha \notin Cn(B')$ but $\neg\alpha \in Cn(B \cup \{\beta\})$ (relevance)

(B*4) $\alpha \in B \mp \alpha$ (success)

(B*6) If α and β are elements of B and it holds for all $B' \subseteq B$ that $\neg\alpha \in Cn(B')$ if and only if $\neg\beta \in Cn(B')$, then $B \cap (B \pm \alpha) = B \cap (B \pm \beta)$ (weak uniformity)

(B*7) $B + \alpha \pm \alpha = B \pm \alpha$ (pre-expansion)

Internal Kernel Revision

(B*1) If $\neg\alpha \notin Cn(\emptyset)$, then $\neg\alpha \notin Cn(B \mp \alpha)$ (non-contradiction)

(B*2) $B \mp \alpha \subseteq B \cup \{\alpha\}$ (inclusion)

(B*3') If $\beta \in B \setminus B \mp \alpha$, then there is some $B' \subseteq B$ such that $\neg\alpha \notin Cn(B')$ and $\neg\alpha \in Cn(B' \cup \{\beta\})$ (core-retainment)

(B*4) $\alpha \in B \mp \alpha$ (success)

(B*5) If for all $B' \subseteq B$, $\neg\alpha \in Cn(B')$ if and only if $\neg\beta \in Cn(B')$, then $B \cap (B \mp \alpha) = B \cap (B \mp \beta)$ (uniformity)

External Kernel Revision

(B*1) If $\neg\alpha \notin Cn(\emptyset)$, then $\neg\alpha \notin Cn(B \mp \alpha)$ (non-contradiction)

(B*2) $B \mp \alpha \subseteq B \cup \{\alpha\}$ (inclusion)

(B*3') If $\beta \in B \setminus B \mp \alpha$, then there is some $B' \subseteq B$ such that $\neg\alpha \notin Cn(B')$ and $\neg\alpha \in Cn(B' \cup \{\beta\})$ (core-retainment)

(B*4) $\alpha \in B \mp \alpha$ (success)

(B*6) If α and β are elements of B and it holds for all $B' \subseteq B$ that $\neg\alpha \in Cn(B')$ if and only if $\neg\beta \in Cn(B')$, then $B \cap (B \pm \alpha) = B \cap (B \pm \beta)$ (weak uniformity)

(B*7) $B + \alpha \pm \alpha = B \pm \alpha$ (pre-expansion)

Two New Operations

Consolidation Makes a belief base consistent.

$B! = B \dot{-} \bot$

Semi-Revision Non-prioritized revision, new info may be rejected.

$B?\alpha = (B + \alpha)!$

- Consolidation possible for bases because information is not lost in the presence of inconsistency.
- Different inconsistent bases react differently to the contraction by falsum.

Lecture 3

New Trends in Revision

Topics:

- Non-prioritized revision.
- Relevance-based revision.
- Approximate belief revision.

Non-prioritized Revision

- Input does not have highest priority and may be rejected.
- Decision + Revision:
  - Decide whether input should be accepted or rejected.
  - If accepted, then revise.
- Expansion + Consolidation:
  - Expand by the input.
  - Make expanded belief state consistent.
- Integrated Choice:
  - Choose (in one step) among the original beliefs and the input.

Screened Revision

(Makinson, 1997)

$$K\#\alpha = \begin{cases} K *_A \alpha & \textit{if } \alpha \textit{ is consistent with } A \cap K \\ K & \textit{otherwise} \end{cases}$$

where A is some set of sentences and $A \cap K \subseteq K *_A \alpha$

- Decide whether α should be accepted.
- Revise by α.

Selective Revision

(Fermé and Hansson, 1998)

In some cases, parts of the input may be acccepted while other parts are rejected.

$$K?\alpha = K * f(\alpha)$$

where f is a transformation function.

Usually, $\alpha \vdash f(\alpha)$.

Using Relevance

Main idea: Not all of the agent's belief are relevant.

Example: I believe it is always raining in Holland.
I see it is a sunny day in Amsterdam $\Rightarrow$ revision

The revision can be done without checking consistency with irrelevant beliefs.

Example

Suppose I am at home and I hear on the radio that my friend Carol has been murdered yesterday night and that there were no traces of doors or windows having been forced. I talked to her yesterday on the phone and she was home with her flatmates Ann and Bill. I know that no one else, except for Ann, Bill and Carol has the keys to their apartment. I conclude that **Ann or Bill must have done it**. But I have known Ann for quite some time and cannot believe that she would be able to murder anyone. I believe **Ann did not do it**. For similar reasons, I believe **Bill did not do it**. This is clearly inconsistent with my belief that one of them did it. So I decide to visit my friend Paul to ask what he thinks. In front of his place I see **the lights are on**. I know that **if the lights are on, then Paul is home**. I get out of the car and Paul's neighbor, inferring that I am coming to visit him tells me **Paul is not home**. This is all very confusing, but I am sure of one thing: I do not believe **I am asleep**!

Compartments

A compartment of B = formulas of B that contribute for (dis)proving some formula in A.

Formally: (C inference operation)

- $B \perp\!\!\!\perp_C \alpha$ = minimal subsets X of B such that $\alpha \in C(B)$
- $c(\alpha, B) = \bigcup((B \perp\!\!\!\perp \alpha) \cup (B \perp\!\!\!\perp \neg\alpha) \setminus (B \perp\!\!\!\perp \perp))$
- $c(A, B) = \bigcup_{\alpha \in A} c(\alpha, B)$

Local Inference

$C_A(B) = Cn(c(A, B))$

$C_A(B)$ gives the classical consequences of those beliefs in B which are relevant for the formulas in A.

- C_A satisfies:
  - $B \subseteq B' \Rightarrow C_A(B) \subseteq C_A(B')$ (monotony)
  - $\alpha \in C_A(B) \Rightarrow$ there is $B' \subset B$, B' finite, s.t. $\alpha \in C_A(B')$ (compactness)
  - $C_A(C_A(B)) = C_A(B)$ (iteration)
- C_A does not satisfy
  - $B \not\subseteq C_A(B)$ (inclusion)

Example revisited

Let p stand for the proposition "Paul is at home",
q for "The lights are on",
a for "Ann is the murderer",
b for "Bill is the murderer" and
r for "I am asleep".

My belief base B after talking to Paul's neighbor contains:

$$\{\neg p, q, q \to p, a \vee b, \neg a, \neg b, \neg r\}.$$

I am interested in whether Paul is at home, that is, the relevant beliefs are:

$$c(p, B) = \{q, q \to p, \neg p\}.$$

Even though this set is inconsistent we have that:

$$r \notin C_r(B) = Cn(c(r, B)) = Cn(\{\neg r\}).$$

Local Contraction

The local partial meet contraction operator based on a local inference operator C_A and a selection function γ is the operator $\dot{-}_{C_A,\gamma}$ such that:

$B \dot{-}_{C_A,\gamma} \alpha = \bigcap \gamma(B \perp_{C_A} \alpha)$.

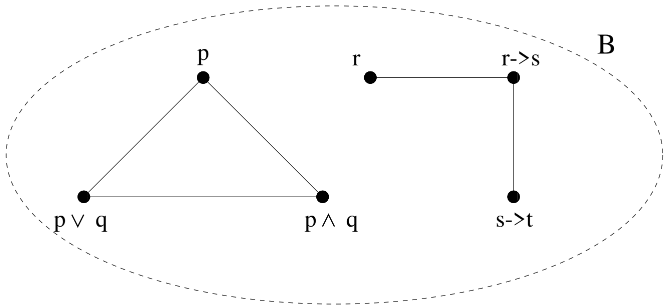
Postulates:

- If $\alpha \notin C_A(\emptyset)$, then $\alpha \notin C_A(B \dot{-} \alpha)$ (success)
- $B \dot{-} \alpha \subseteq B$ (inclusion)
- If $\beta \in B \setminus (B \dot{-} \alpha)$, then there is some B' such that $B \dot{-} \alpha \subseteq B' \subseteq B$, $\alpha \notin C_A(B')$ and $\alpha \in C_A(B' \cup \{\beta\})$ (relevance)
- If for all subsets B' of B, $\alpha \in C_A(B')$ if and only if $\beta \in C_A(B')$, then $B \dot{-} \alpha = B \dot{-} \beta$ (uniformity)

Structured Bases

- Problems:
  - Finding the compartments is very expensive.
  - There is no guaranty that the compartment will be small.
- But: Representation results do not depend on the way the compartments are found!
- So: Look for more efficient method.
- Idea: Organize belief base as a graph, where links represent relevance.

Relevance Graph



$\mathcal{R}(\varphi, \psi)$ if and only if φ and ψ share an atom

Alternative for Compartments

Definition: Formulas of B relevant for α up to degree n:

$\Delta^n(\alpha, B) = \{\varphi \in B | d(\alpha, \varphi) \leq n\}$

Set of relevant formulas for α: $\Delta^{\leq\omega}(\alpha, B) = \bigcup_{i \geq 0} \Delta^i(\alpha, B)$

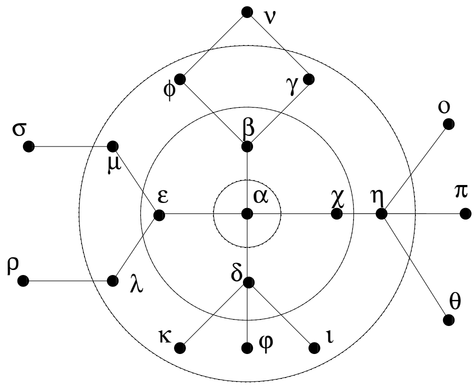
Proposition: Let B be a belief base and α a formula. The local inference operations $C^n_\alpha(B) = Cn(\Delta^{\leq n}(\alpha, B))$ are compact and monotonic.

So: $\Delta^{\leq n}$ can be used for local change.

Computational Aspects – 1

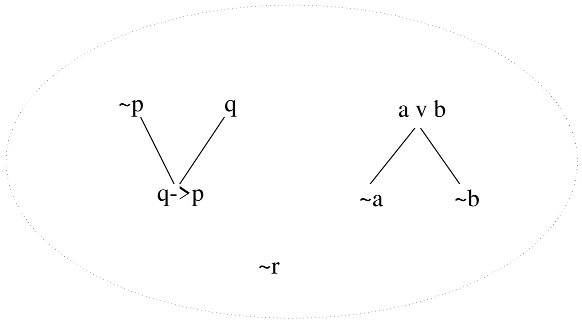
- Retrieval can be done in linear time.
- Anytime method: anytime the algorithm is interrupted, it returns the most relevant beliefs.
- Ideal case - the algorithm returns the connected subgraph $\Delta^{\leq\omega}(\alpha, B)$

Computational Aspects – 2



Example re-revisited

My belief base B after talking to Paul's neighbor contains:



Example re-revisited – 2

I am interested in whether Paul is at home, that is, the relevant beliefs are:

$$\Delta^{\leq\omega}(p, B) = \{q, q \to p, \neg p\}.$$

Even though this set is inconsistent we have that:

$$r \notin C^{\omega}_{r}(B) = Cn(\Delta^{\leq\omega}(r, B)) = Cn(\{\neg r\}).$$

Approximate Entailment – Cadoli and Schaerf

Main idea: Consider only some propositional letters.

- L: propositional letters of the language.
- Context set $S \subseteq L$.
- Define two approximate entailments:
  - $\models^1_S$: complete but not sound.
  - $\models^3_S$: sound and incomplete.
- $S = L$: classical entailment.
- Approximations behave classically for $x \in S$.
  - S_1 assignment: if $x \notin S$, then $x, \neg x$ both false.
  - S_3 assignment: if $x \notin S$, then $x, \neg x$ not both false.

Example – Cadoli and Schaerf

$$\begin{aligned} B = \; & \{\neg \mathtt{cow} \lor \mathtt{grass\text{-}eater}, \neg \mathtt{dog} \lor \mathtt{carnivore}, \\ & \neg \mathtt{grass\text{-}eater} \lor \neg \mathtt{has\text{-}canine\text{-}teeth}, \\ & \neg \mathtt{grass\text{-}eater} \lor \mathtt{mammal}, \\ & \neg \mathtt{carnivore} \lor \mathtt{mammal}, \\ & \neg \mathtt{mammal} \lor \mathtt{has\text{-}canine\text{-}teeth} \lor \mathtt{has\text{-}molar\text{-}teeth}, \\ & \neg \mathtt{mammal} \lor \mathtt{vertebrate}, \\ & \neg \mathtt{vertebrate} \lor \mathtt{animal}\}. \end{aligned}$$

Let $\alpha = \neg \mathtt{cow} \lor \mathtt{has\text{-}molar\text{-}teeth}$.

$B \models \alpha$?

For $S = \{\mathtt{grass\text{-}eater}, \mathtt{mammal}, \mathtt{has\text{-}canine\text{-}teeth}\}$,

$B \models^3_S \alpha$, hence $B \models \alpha$.

Approximate Belief Revision

(Chopra, Parikh, Wassermann, WOLLIC 2000)

- Goal:
  - Local change.
  - Inconsistency tolerance.
  - Computational feasibility.
  - Logical elegance.
- Proposal: Integrate approximate inference relations with Local Change model.

Recap: Local Contraction

- Definition: Let C be an inference operation on L and σ an incision function. The local kernel contraction of B determined by C and σ is the operation $\dot{-}_{C,\sigma}$ such that $\forall B$: $B \dot{-}_{C,\sigma} \alpha = B \setminus \sigma(B \perp\!\!\!\perp_C \alpha)$
- Theorem: Let C be an inference operation satisfying monotonicity and compactness. Then $\dot{-}$ is an operation of local kernel contraction determined by C and some incision function iff $\forall B$:
  - If $\alpha \notin C(\emptyset)$, then $\alpha \notin C(B \dot{-} \alpha)$ (success)
  - $B \dot{-} \alpha \subseteq B$ (inclusion)
  - If $x \in B \setminus B \dot{-} \alpha$, then there is some $B' \subseteq B$ such that $\alpha \notin C(B')$ and $\alpha \in C(B' \cup \{x\})$ (core-retainment)
  - If for all subsets B' of B $\alpha \in C(B')$ if and only if $\beta \in C(B')$, then $B \dot{-} \alpha = B \dot{-} \beta$ (uniformity)

Approximating Belief Change

- $C_S^i(B) = \{\alpha | B \models_S^i \alpha\}$
- Proposition: Let S be a fixed set of propositional variables. Then C_S^i satisfies monotony and compactness.
- Corollary: C_S^i can be used for local change.
- Approximate kernel contraction of B determined by C_S^i:
  - $B \dot{-}_{C_S^i,\sigma} \alpha = B \setminus \sigma(B \perp\!\!\perp_{C_S^i} \alpha)$
- **Result:** two kinds of contraction operations:
  - one more radical (based on S_1)
  - one more cautious (based on S_3)

Problem: Choice of S

- Suggestion: Use some notion of relevance.
- For example, syntactic relevance:
- $R(p,q,B)$ iff p and q appear in the same formula of B.

  Construction strategy for S:
  - Start with $L(\alpha)$.
  - Test entailments.
  - If the result is not satisfactory and resources are available, add to S the atoms relevant to elements in S and go to 2.
  - Return approximations.

Example – Hans the student

$B =$ {student , ¬student ∨ young,
¬young ∨ ¬pensioner, worker,
¬worker ∨ ¬pensioner, blue-eyes,
likes-dancing, six-feet-tall}.

- Start with S={pensioner}:
  - $B \perp\!\!\perp_{C_S^3}$ ¬pensioner= ∅.
- S= {pensioner, young, worker}
  - $B \perp\!\!\perp_{C_S^3}$¬pensioner = {{worker, ¬worker ∨ ¬pensioner}}.
- S={pensioner, young, worker, student}
  - $B \perp\!\!\perp_{C_S^3}$¬pensioner={{student, ¬student ∨ young, ¬young ∨ ¬pensioner}, {worker, ¬worker ∨ ¬pensioner}}.

To Conclude

What have we done?

- Started from classical AGM theory
- Proceeded in the direction of more applicable methods:
  - Belief Bases
  - Models based on relevance
  - Approximations

Course Outline

- **Lecture 1:** AGM Theory
- **Lecture 2:** Base Dynamics
- **Lecture 3:** New Trends in Revision
- **Lecture 4:** The theory of update
- **Lecture 5:** Computational models of update

Lecture 4

The Theory of Update

material from (Herzig 1998): **revised** and **updated**

- updates vs. revisions
- world-wise change
- updates vs. conditionals: the Ramsey test
- logics based on the Ramsey test

Is every belief change of AGM type?

- beliefs

  The window is open or the door is open.
- new information

  Somebody closes the door.
- belief change according to AGM

  The window is open.

Revision does not apply here!

Where does AGM-revision apply?

- scientific theories (Gärdenfors, Segerberg)
  - ‘theory revision’
  - small changes only (no paradigm shifts)
- regulations (Alchourrón)
  - derogations in legal systems
- diagnosis
- commonsense reasoning???
- databases???
- reasoning about actions and events???

Two distinct operations

(Keller & Winslett 1985)

- beliefs

  I believe Kaurismäki lives in Finland.

- What changes?
  1. recording of change in the world:

     I learn that he has just moved to Hollywood.

     ▶ update
  2. new information about an unchanged world:

     I learn that I have been mistaken and that he lives in Hollywood.

     ▶ revision

World-wise change – the idea

(Katsuno & Mendelzon, KR 1991)

- interpretation of a belief set B

  = set of possible worlds where B is true
  (may contain the real world, but need not)
- notification of some change in the real world
- The description of the possible states of affairs must be modified accordingly:

  > *Since we are confined to our set of possibilities, we must make the change come true in all of our candidate worlds. Semantically, we change each of the possible worlds 'as little as possible' in order to make the new state of affairs hold. (Grahne, KR 1991)*

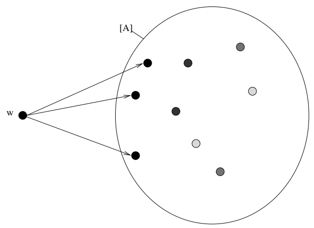
models?

World-wise change – selection functions

- interpretation of $B = \{w | w \text{ is a possible world, and } w \models B\}$
  $= [B]$
- update of a world $w \in [B]$ by A

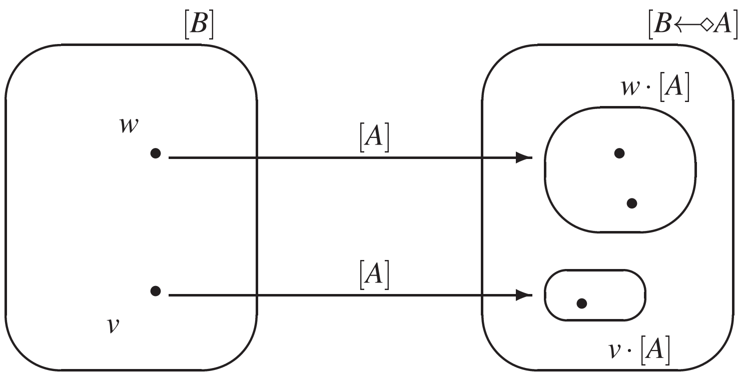
  = the possible worlds close to w where A is true

  = $w \cdot [A]$ (selection function)



Selection function semantics

update of B by A $= [B \leftarrow\!\diamond A] = \bigcup_{w \in [B]} w \cdot [A]$



postulates? properties? constructions?

Updates vs. conditionals

- update problem: given
  - current belief base B,
  - input A,
  - a belief C,

  does C result from updating B by A?
- ternary relation $\mathcal{R}(B,A,C)$; algebraically:
  1. $\mathcal{R}(B,A,C)$ iff $B \leftarrow\!\diamond A \vdash C$
     $B \leftarrow\!\diamond A$ "B has been updated by A"
     'looking back'; used in databases
  2. $\mathcal{R}(B,A,C)$ iff $B \vdash A \Box\!\!\rightarrow C$
     $A \Box\!\!\rightarrow C$ "if one updates with A then C follows"
     'looking forward'; studied in philosophical logic

Updates vs. conditionals – the Ramsey test

$$B \leftarrow\!\diamond A \vdash C \text{ iff } B \vdash A \Box\!\!\rightarrow C$$

If two people are arguing 'If p will q and are both in doubt as to p, they are adding p hypothetically to their stock of knowledge and arguing on that basis about q. *(Ramsey 1931)*

This is how to evaluate a conditional: First, add the antecedent (hypothetically) to your stock of beliefs; second, make whatever adjustments are required to maintain consistency (without modifying the hypothetical belief in the antecedent); finally, consider whether or not the consequent is then true. *(Stalnaker 1968)*

Updates vs. conditionals: Gärdenfors' impossibility theorem – 1

(Gärdenfors 1978)

Theorem: Suppose

- $\leftarrow\!\diamond$ is a metalanguage belief change operation (mapping bases and inputs to new bases)
- $\mathcal{L}$ is a logic for an (object language) $\Box\!\!\rightarrow$
- $\leftarrow\!\diamond$ and $\Box\!\!\rightarrow$ satisfy the Ramsey test:

$$B \leftarrow\!\diamond A \vdash C \text{ iff } \vdash_{\mathcal{L}} B \rightarrow (A \Box\!\!\rightarrow C)$$

- the logic is 'rich enough'

Then $\leftarrow\!\diamond$ does not satisfy the AGM postulates.

Gärdenfors' impossibility theorem – 2

Let $A_1 \wedge A_2$, $\neg(A_1 \wedge A_2)$, etc. be all consistent. (L is rich enough.)

Let B be ignorant w.r.t. A_1 and A_2, i.e. $B \not\vdash A_1$, $B \not\vdash \neg A_1$, $B \not\vdash A_2$, $B \not\vdash \neg A_2$.

Suppose $\leftarrow\!\diamond$ satisfies the AGM postulates, and let
$B_1 = B \leftarrow\!\diamond A_1$, $B_2 = B \leftarrow\!\diamond A_2$, $B_{12} = B \leftarrow\!\diamond (A_1 \wedge A_2)$.

1. $B_1 \leftrightarrow B \wedge A_1$, and $B_2 \leftrightarrow B \wedge A_2$ — because $\leftarrow\!\diamond$ satisfies AGM
2. $B_1 \leftarrow\!\diamond \neg(A_1 \wedge A_2) \leftrightarrow B \wedge A_1 \wedge \neg A_2$, and $B_2 \leftarrow\!\diamond \neg(A_1 \wedge A_2) \leftrightarrow B \wedge \neg A_1 \wedge A_2$ — again because $\leftarrow\!\diamond$ satisfies AGM
3. $\vdash_{\mathcal{L}} B_1 \rightarrow \neg(A_1 \wedge A_2) \Box\!\!\rightarrow A_1$, and $\vdash_{\mathcal{L}} B_2 \rightarrow \neg(A_1 \wedge A_2) \Box\!\!\rightarrow A_2$ — by the Ramsey test
4. $B_{12} \leftrightarrow B \wedge A_1 \wedge A_2$ — because $\leftarrow\!\diamond$ satisfies AGM
5. $B_{12} \rightarrow B_1$ and $B_{12} \rightarrow B_2$ — with 1
6. $\vdash_{\mathcal{L}} B_{12} \rightarrow (\neg(A_1 \wedge A_2) \Box\!\!\rightarrow A_1)$, and $\vdash_{\mathcal{L}} B_{12} \rightarrow (\neg(A_1 \wedge A_2) \Box\!\!\rightarrow A_2)$ — with 3

Gärdenfors' impossibility theorem – 3

7. $(B_{12} \leftarrow\!\!\diamond \neg(A_1 \wedge A_2)) \rightarrow A_1$, and $(B_{12} \leftarrow\!\!\diamond \neg(A_1 \wedge A_2)) \rightarrow A_2$ again by the Ramsey test
8. $B_{12} \leftarrow\!\!\diamond \neg(A_1 \wedge A_2) \rightarrow (A_1 \wedge A_2)$ from 7
9. $B_{12} \leftarrow\!\!\diamond \neg(A_1 \wedge A_2) \rightarrow \perp$ because $\leftarrow\!\!\diamond$ satisfies AGM
10. ...but $B_{12} \leftarrow\!\!\diamond \neg(A_1 \wedge A_2)$ should be consistent (because $A_1 \wedge A_2$ is consistent)

Remarks:

1. Gärdenfors' result does not depend on $\vdash$, and the theorem holds even if $\vdash$ is a substructural or nonmonotonic logic (Makinson, SL 1990)
2. similar to Gödel's incompleteness result: no object language predicate can express consistency
3. does not hold for updates (Grahne, KR 1991)

Rest of lecture 4:

Logics based on the Ramsey test

- language vs. metalanguage
- taking the Ramsey test seriously
- some derivable principles
- normal update logics
- cumulative update logics
- the KM-theory as an extension

Language vs. metalanguage

- traditionally:
  - $\leftarrow\!\!\diamond$ metalanguage operation
    - single model
    - non-constructive postulates
      "if A consistent then $B \leftarrow\!\!\diamond A$ consistent", ...
  - $\leftarrow\!\!\diamond$ object language operator
    - classes of models
    - axiomatics (constructive)
- here:
  - two dyadic modal operators in the object language
    - update operator: $\leftarrow\!\!\diamond$
    - conditional operator: $\Box\!\!\rightarrow$

Taking the Ramsey test seriously

- classical logic, plus
- ...just two inference rules: "the Ramsey rules"
  - $\dfrac{B \leftarrow\!\!\diamond A \rightarrow C}{B \rightarrow A \Box\!\!\rightarrow C}$
  - $\dfrac{B \rightarrow A \Box\!\!\rightarrow C}{B \leftarrow\!\!\diamond A \rightarrow C}$

Updates and conditionals as inverse modal operators

(Ryan & Schobbens, JoLLI 1997)

Switching Lemma (de Rijke, Venema, Roorda, Dunn):

- The conversion axioms
  - $B \rightarrow A \Box\!\!\rightarrow (B \leftarrow\!\!\diamond A)$
  - $(A \Box\!\!\rightarrow C) \leftarrow\!\!\diamond A \rightarrow C$

  can be derived from the Ramsey rules
- ... and vice versa

cf. tense logics, dynamic logic with inverse modality α^{-1}, program logics:

$A \Box\!\!\rightarrow C = \overrightarrow{\Box}_A\, C$

$B \leftarrow\!\!\diamond A = \overleftarrow{\Diamond}_A\, B$

Derivable principles for updates – 1

- syntax-independence
  $\dfrac{B_1 \leftrightarrow B_2}{B_1 \leftarrow\!\!\diamond A \leftrightarrow B_2 \leftarrow\!\!\diamond A}$
- monotony
  $\dfrac{B_1 \rightarrow B_2}{B_1 \leftarrow\!\!\diamond A \rightarrow B_2 \leftarrow\!\!\diamond A}$
- updating disjunctions
  $(B_1 \leftarrow\!\!\diamond A \vee B_2 \leftarrow\!\!\diamond A) \leftrightarrow (B_1 \vee B_2) \leftarrow\!\!\diamond A$
- inconsistency preservation
  $\bot \leftarrow\!\!\diamond A \rightarrow \bot$

Derivable principles for updates – 2

- example: proof of $\bot \leftarrow\!\!\diamond A \rightarrow \bot$

  1. $\bot \rightarrow A \Box\!\!\rightarrow \bot$ — by classical logic
  2. $\bot \leftarrow\!\!\diamond A \rightarrow \bot$ — by the Ramsey rules

- example: proof of $\dfrac{B_1 \equiv B_2}{B_1 \leftarrow\!\!\diamond A \equiv B_2 \leftarrow\!\!\diamond A}$

  1. $B_1 \equiv B_2$ — hypothesis
  2. $\dfrac{B_1 \rightarrow A \Box\!\!\rightarrow C}{B_2 \rightarrow A \Box\!\!\rightarrow C}$ — from 1, with fresh C
  3. $\dfrac{B_1 \leftarrow\!\!\diamond A \rightarrow C}{B_2 \leftarrow\!\!\diamond A \rightarrow C}$ — from 2, by the Ramsey rules
  4. $B_2 \leftarrow\!\!\diamond A \rightarrow B_1 \leftarrow\!\!\diamond A$ — from 3
  5. $B_1 \leftarrow\!\!\diamond A \rightarrow B_2 \leftarrow\!\!\diamond A$ — with a proof similar to 1–4

Derivable principles for conditionals

(Wansing, JLC 1994)

- $\dfrac{A_1 \leftrightarrow A_2}{A_1 \Box\!\!\rightarrow C \leftrightarrow A_2 \Box\!\!\rightarrow C}$
- $\dfrac{C_1 \leftrightarrow C_2}{A \Box\!\!\rightarrow C_1 \leftrightarrow A \Box\!\!\rightarrow C_2}$
- $\dfrac{C_1 \rightarrow C_2}{A \Box\!\!\rightarrow C_1 \rightarrow A \Box\!\!\rightarrow C_2}$
- $A \Box\!\!\rightarrow (C_1 \wedge C_2) \leftrightarrow (A \Box\!\!\rightarrow C_1 \wedge A \Box\!\!\rightarrow C_2)$
- $A \Box\!\!\rightarrow \top$

axioms of semi-normal conditional logic Ck (Chellas, JPL 1975)

Recovering normal conditional logics

CK (C "conditional", K "Kripke")

$$= \text{Ck} + \frac{A_1 \leftrightarrow A_2}{A_1 \Box\!\!\rightarrow C \leftrightarrow A_2 \Box\!\!\rightarrow C}$$

- selection function semantics:
  - $M = \langle W, \cdot, V \rangle$ such that
    - W (possible worlds)
    - $\cdot : W \times 2^W \longrightarrow 2^W$ (selection function)
    - $V(w) \subseteq ATM$ for all $w \in W$ (truth assignment)
  - $[A \Box\!\!\rightarrow C] = \{w \in W | w \cdot [A] \subseteq [C]\}$

Theorem (Chellas, JPL 1975): sound and complete, finite model property, decidable

The other way round:

- given
  - a conditional logic $\mathcal{L}_{\Box\!\rightarrow}$
  - its tense extension $\mathcal{L}_{\Box\!\rightarrow, \leftarrow\!\diamond}$ by the Ramsey rules
- which properties does $\mathcal{L}_{\Box\!\rightarrow}$ inherit? (completeness, decidability, finite model, correspondence)
- ALL if $\mathcal{L}$ is one of the standard systems
- CORRESPONDENCE ONLY in the general case (Wolter, NDJFL 1998)

Normal update logics

UCK (U "update", C "conditional", K "Kripke")

$= \text{CK} + \text{Ramsey rules}$

$$= \text{classical logic} + \text{Ramsey rules} + \frac{A_1 \leftrightarrow A_2}{A_1 \Box\!\!\rightarrow C \leftrightarrow A_2 \Box\!\!\rightarrow C}$$

$$= \text{classical logic} + \text{Ramsey rules} + \frac{A_1 \leftrightarrow A_2}{B \leftarrow\!\diamond A_1 \leftrightarrow B \leftarrow\!\diamond A_2}$$

Theorem (Herzig 1998): sound and complete for selection function semantics.

$[B \leftarrow\!\diamond A] = \bigcup_{w \in [B]} w \cdot [A]$ $\quad$ $[A \Box\!\!\rightarrow C] = \{w \in W | w \cdot [A] \subseteq [C]\}$

Four desiderata for belief update

- syntax-independence
  - if $A_1 \leftrightarrow A_2$ then $B \leftarrow\!\diamond A_1$ equivalent to $B \leftarrow\!\diamond A_2$
  - if $B_1 \leftrightarrow B_2$ then $B_1 \leftarrow\!\diamond A$ equivalent to $B_2 \leftarrow\!\diamond A$
- success
  - $B \leftarrow\!\diamond A \vdash A$
- consistency 'as much as possible'
  - $B \leftarrow\!\diamond A$ is consistent iff both B and A are consistent.
- minimal change
  - $B \leftarrow\!\diamond A$ is 'as close to B as possible'

the latter 3 require extensions of UCK . . .

UCK and the desiderata – 1

- success
  - axiomatically:
    UCK $+ B \leftarrow\!\!\diamond A \;\rightarrow\; A$ alternative: $+ A \Box\!\!\rightarrow A$
  - semantically: $w \cdot [A] \subseteq [A]$
- consistency ‘as much as possible’
  - axiomatically:
    UCK $+ \dfrac{B \leftarrow\!\!\diamond A \rightarrow \bot}{B \rightarrow \bot}$ alternative: $+ \neg(A \Box\!\!\rightarrow \bot)$
  - semantically: $w \cdot [A] \neq \emptyset$, for all $w \in W$

  for every consistent formula of classical logic A

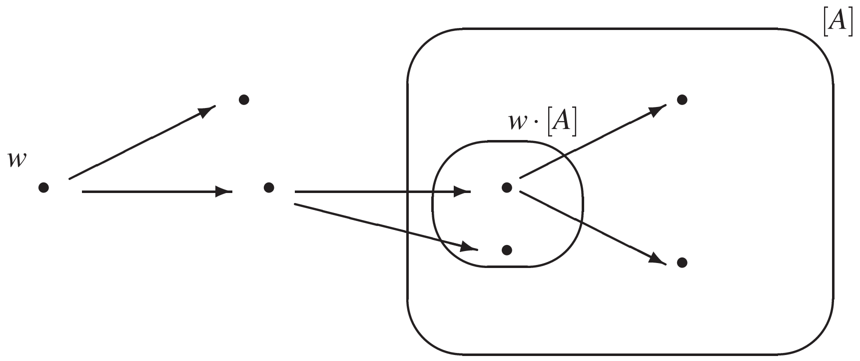
UCK and the desiderata – 2

- minimal change
  - axiomatically: difficult to express if $B \wedge A$ is inconsistent
  - semantically: minimal change ‘defined away’ by selection function
- but weaker forms of minimal change
  - UCK $+ \dfrac{B \rightarrow A}{B \leftarrow\!\!\diamond A \leftrightarrow B}$ (idempotence)
  - ...

minimality $\sim$ orderings ...

Cumulative update logics

minimal change: orderings of closeness associated to worlds



Cumulative update logics – semantics

- $M = \langle W, \{\leq_w \mid w \in W\}, V\rangle$ such that
  - W some set (possible worlds)
  - $\leq_w$ partial pre-order on W (reflexive and transitive relation)
    $u \leq_w v$ = “u is at least as close to w as v is”
  - $V(w)$ truth assignment
- $S_w = \{u \in W \mid$ there is $v \in W$ such that $u \leq_w v\}$ (accessible worlds)
- $w \cdot [A]$ = the A-worlds closest to w
  $= \min_{\leq_w}([A] \cap S_w)$
  $= \{u \in [A] \cap S_w \mid$ for all $v \in [A] \cap S_w$, if $v \leq_w u$ then $u \leq_w v\}$
- truth conditions as for UCK
- if W infinite: Limit Assumption; guarantees that $[A] \neq \emptyset$ implies $w \cdot [A] \neq \emptyset$

Cumulative conditional logics – axiomatics

CL (C "conditional", L "Lewis")

$= \text{CK}$

$+\ A \Box\!\!\rightarrow A$

$+\ ((A_1 \Box\!\!\rightarrow C) \wedge (A_2 \Box\!\!\rightarrow C)) \ \rightarrow\ ((A_1 \vee A_2) \Box\!\!\rightarrow C)$

$+\ ((A_1 \Box\!\!\rightarrow A_2) \wedge (A_2 \Box\!\!\rightarrow A_1)) \ \rightarrow\ ((A_1 \Box\!\!\rightarrow C) \leftrightarrow (A_2 \Box\!\!\rightarrow C))$

(Lewis 1973, Burgess 1981)

Cumulative update logics – axiomatics

UCL (U "update", C "conditional", L "Lewis")

$= \text{UCK} + \text{CL}$

$= \text{UCK}$

$+\ (B \leftarrow\!\!\diamond A) \ \rightarrow\ A$

$+\ (B \leftarrow\!\!\diamond (A_1 \vee A_2)) \ \rightarrow\ ((B \leftarrow\!\!\diamond A_1) \vee (B \leftarrow\!\!\diamond A_2))$

$+\ \dfrac{(B \leftarrow\!\!\diamond A_1) \rightarrow A_2\ ,\ (B \leftarrow\!\!\diamond A_2) \rightarrow A_1}{(B \leftarrow\!\!\diamond A_1) \leftrightarrow (B \leftarrow\!\!\diamond A_2)}$

UCL: derivable principles

- cumulativity
  - $(A_1 \Box\!\!\rightarrow A_2) \ \rightarrow\ ((A_1 \Box\!\!\rightarrow C) \leftrightarrow (A_2 \Box\!\!\rightarrow C))$
  - $\dfrac{B \leftarrow\!\!\diamond A_1 \rightarrow A_2}{(B \leftarrow\!\!\diamond A_1) \leftrightarrow (B \leftarrow\!\!\diamond A_1 \wedge A_2))}$
- ...

KM theory

(**K**atsuno & **M**endelzon, KR 1991)

- A KM-model is a UCL-model $M = \langle W, \{ \leq_w | w \in W \}, V \rangle$ where
  - $W = 2^{ATM}$ the set of **all** logically possible worlds (= all truth assignments)
  - $\leq_w$ is a 'faithful' partial pre-order:
    - $w \leq_w u$ for all u
    - if $u \leq_w w$ then $u = w$

    consequence: $\min_{\leq_w}(W) = \{w\}$
  - $V(w) = w$ (every w is a set of atoms)

The KM-postulates

(U1) $B \leftarrow\!\diamond A \ \rightarrow\ A$

(U2) If $B\ \rightarrow\ A$ then $B \leftarrow\!\diamond A\ \leftrightarrow\ B$

(U3) If B and A are consistent, then $B \leftarrow\!\diamond A$ is consistent.

(U4.1) If $B_1\ \leftrightarrow\ B_2$ then $B_1 \leftarrow\!\diamond A\ \leftrightarrow\ B_2 \leftarrow\!\diamond A$

(U4.2) If $A_1\ \leftrightarrow\ A_2$ then $B \leftarrow\!\diamond A_1\ \leftrightarrow\ B \leftarrow\!\diamond A_2$

(U5) $(B \leftarrow\!\diamond A) \wedge C\ \rightarrow\ B \leftarrow\!\diamond (A \wedge C)$

(U6) If $(B \leftarrow\!\diamond A_1) \rightarrow A_2$ and $(B \leftarrow\!\diamond A_2) \rightarrow A_1$ then $B \leftarrow\!\diamond A_1 \leftrightarrow B \leftarrow\!\diamond A_2$

(U7) $(B \leftarrow\!\diamond A_1) \wedge (B \leftarrow\!\diamond A_2)\ \rightarrow\ B \leftarrow\!\diamond (A_1 \vee A_2)$
if B is complete (requires finite language!)

(U8) $(B_1 \vee B_2) \leftarrow\!\diamond A\ \leftrightarrow\ (B_1 \leftarrow\!\diamond A) \vee (B_2 \leftarrow\!\diamond A)$

Theorem: sound and complete for finite languages

Lecture 5

Computational Models of Update

material from (Herzig & Rifi, AIJ 1999), with **minimal changes** only ...

- concrete update operations
- handling of disjunctive input
- status of the KM-postulates
- handling of integrity constraints

KM theory and the desiderata

- success = (U1)
- syntax-independence = (U4.1) + (U4.2),
- consistency 'as much as possible' = (U3)
- minimal change
  ... not always satisfied

KM-theory and minimal change

- faithful partial pre-order for maximal change:

$$\leq_w\ = \{\langle w, u\rangle | u \in W\}$$

- consequence:

$$\min_{\leq_w}([A]) = \begin{cases} \{w\} & \text{if } w \in [A] \\ [A] & \text{else} \end{cases}$$

- the maximal change operation

$$B \leftarrow\!\diamond A \leftrightarrow \begin{cases} B \text{ if } B \vdash A \\ A \text{ if } B \not\vdash A \end{cases}$$

is a KM-update operation!

Minimal change: intuitions

- exclude the maximal change pre-order
- focus on orderings such that

$$\emptyset \leq_\emptyset \{p\} \leq_\emptyset \{p,q\}$$

($\{p\}$ closer to $\emptyset$ than $\{p,q\}$)

- guarantees $(\neg p \land \neg q) \leftarrow\!\!\diamond p \;\rightarrow\; \neg q$

unique orderings ...

Forbus' semantics – 1

(Forbus, IJCAI 1989)

- particular KM closeness relation $\leq_w$
- unique model
- distance-based:

$$dist_F(w,u) = \text{the number of atoms whose truth value differs}$$

- $dist_F(\emptyset,\{p\}) = 1$
  $dist_F(\emptyset,\{p,q\}) = 2$
  $dist_F(\{p\},\{q\}) = 2$
- revision counterpart: (Dalal, AAAI 1988)

Forbus' semantics – 2

- $dist_F$ induces faithful total pre-order:
  $u \leq_w v$ iff $dist_F(w,u) \leq dist_F(w,v)$
- $\emptyset <_\emptyset \{p\} <_\emptyset \{p,q\}$
- $w \cdot [A] = \min_{\leq_w}([A])$
  $= \{u \mid$ there is no u' such that $dist_F(w,u') < dist_F(w,v)$
- $(\neg p \land \neg q) \leftarrow\!\!\diamond p \;\leftrightarrow\; (p \land \neg q)$

- too strong?

$$(\neg p_1 \land \neg p_2 \land \neg p_3) \leftarrow\!\!\diamond (p_1 \lor (p_2 \land p_3)) \;\leftrightarrow\; (p_1 \land \neg p_2 \land \neg p_3)$$

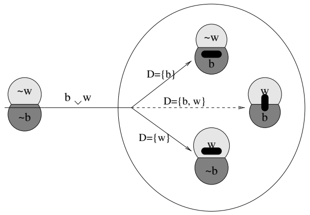
The Possible Models Approach (*PMA*)

(Winslett, AAAI 1988)

- $dist(w,u)$ = the set of atoms whose truth value differs
  $dist(\emptyset,\{p_1\}) = \{p_1\}$
  $dist(\emptyset,\{p_2,p_3\}) = \{p_2,p_3\}$
  $dist(\{p_2\},\{p_3\}) = \{p_2,p_3\}$
- $dist$ induces faithful partial pre-order:
  $u \leq_w v$ iff $dist(w,u) \subseteq dist(w,v)$
- $(\neg p_1 \land \neg p_2 \land \neg p_3) \leftarrow\!\!\diamond (p_1 \lor (p_2 \land p_3)) \;\leftrightarrow\; (p_1 \lor (p_2 \land p_3))$

The *PMA*: critiques – 1

- example: throwing a coin on a chessboard [Reiter]



  - $B = \neg Black \wedge \neg White$
  - $A = Black \vee White$
  - $B \longleftarrow\!\!\diamond A \;\rightarrow\; \neg(Black \wedge White)$

The *PMA*: critiques – 2

- another example:
  - I believe Kaurismäki lives in SW Finland.
  - I learn that he does not live in SW Finland any longer.
  - Conclusion: he does not live in NE Finland.

  $(South \wedge West) \longleftarrow\!\!\diamond (\neg South \vee \neg West) \rightarrow (South \vee West)$
- exclusive interpretation of disjunctive updates:

  $$\frac{B \rightarrow \neg(A_1 \vee A_2)}{B \longleftarrow\!\!\diamond (A_1 \vee A_2) \;\rightarrow\; \neg(A_1 \wedge A_2)}$$
- corrections based on minimal distance:
  MCD (Zhang & Foo 1996); *MCD** (Herzig & Rifi 1998)
  …but there are counterexamples

updates cannot be based on minimizing distances

The *PMA*: generalizing the critiques

Theorem (Herzig & Rifi 1998):
Suppose

- $\longleftarrow\!\!\diamond$ satisfies the KM postulates (U1) and (U5);
- A_1 and A_2 are independent: $B \longleftarrow\!\!\diamond A_1 \rightarrow \neg A_2$ and $B \longleftarrow\!\!\diamond A_2 \rightarrow \neg A_1$.

Then

$$B \longleftarrow\!\!\diamond (A_1 \vee A_2) \;\rightarrow\; \neg(A_1 \wedge A_2)$$

- unintuitive property for updates (but rational for revision!)
- KM theory must be abandonned
- updates cannot be based on orderings

Lecture 5, half way

Computational models of update

- KM-semantics, based on a particular ordering
  - *FORBUS*, *PMA*, *MCD*, *MCD**
  - critique: handling of disjunctive input
- back to selection function semantics

Winslett's Standard Semantics (*WSS*)

(Winslett 1990)

- $w \cdot A = \{v \in [A] | dist(w,v) \subseteq atm(A)\}$
- $atm(A)$ = set of atoms occurring in A
- $atm(A)$: set of atoms allowed to change
- 'syntax-based' ($w \cdot A$ instead of $w \cdot [A]$)
- $B{\leftarrow\!\diamond}South \not\leftrightarrow B{\leftarrow\!\diamond}(South \wedge (West \vee \neg West))$

$WSS{\downarrow}$ = *WSS* plus normal forming

(Herzig & Rifi, ECAI 1998)

- eliminate redundant atoms:
  $(p \wedge (q \vee \neg q)){\downarrow} = p$
- p redundant in A iff $A[p := \top] \leftrightarrow A[p := \bot]$
- $A{\downarrow}$: no redundant atoms
- $w \cdot A = \{v \in [A] | dist(w,v) \subseteq atm(A{\downarrow})\}$
- syntax-independent:
  if $A \leftrightarrow A'$ then $w \cdot A = w \cdot A'$
- equivalent proposal: *MPMA* (Doherty et al., KR 1998)
- similar but more conservative: *MCE* (Zhang & Foo 1996)

$WSS{\downarrow}$ and the KM-theory

(U1) $B{\leftarrow\!\diamond}A \rightarrow A$

(U2.1) If $B \rightarrow A$ then $B \rightarrow B{\leftarrow\!\diamond}A$

(U2.2) If $B \rightarrow A$ then $B{\leftarrow\!\diamond}A \rightarrow B$

(U3) If B and A are consistent, then $B{\leftarrow\!\diamond}A$ is consistent.

(U4.1) If $B_1 \leftrightarrow B_2$ then $B_1{\leftarrow\!\diamond}A \leftrightarrow B_2{\leftarrow\!\diamond}A$

(U4.2) If $A_1 \leftrightarrow A_2$ then $B{\leftarrow\!\diamond}A_1 \leftrightarrow B{\leftarrow\!\diamond}A_2$

(U5) $(B{\leftarrow\!\diamond}A) \wedge C \rightarrow B{\leftarrow\!\diamond}(A \wedge C)$

(U6) If $(B{\leftarrow\!\diamond}A_1) \rightarrow A_2$ and $(B{\leftarrow\!\diamond}A_2) \rightarrow A_1$ then $B{\leftarrow\!\diamond}A_1 \leftrightarrow B{\leftarrow\!\diamond}A_2$

(U7) $(B{\leftarrow\!\diamond}A_1) \wedge (B{\leftarrow\!\diamond}A_2) \rightarrow B{\leftarrow\!\diamond}(A_1 \vee A_2)$ if B is complete

(U8) $(B_1 \vee B_2){\leftarrow\!\diamond}A \leftrightarrow (B_1{\leftarrow\!\diamond}A) \vee (B_2{\leftarrow\!\diamond}A)$

(U2.2), (U5), (U6), (U7) violated

$WSS{\downarrow}$: a critique of the violated postulates

(U7) $(B{\leftarrow\!\diamond}A_1) \wedge (B{\leftarrow\!\diamond}A_2) \rightarrow B{\leftarrow\!\diamond}(A_1 \vee A_2)$ if B is complete
'technical' postulate to ensure completenes of KM theory
(makes no sense if language is infinite)

(U6) If $(B{\leftarrow\!\diamond}A_1) \rightarrow A_2$ and $(B{\leftarrow\!\diamond}A_2) \rightarrow A_1$ then $B{\leftarrow\!\diamond}A_1 \leftrightarrow B{\leftarrow\!\diamond}A_2$
entails (U2.2) under $B{\leftarrow\!\diamond}\top \rightarrow B$: hence v. (U2.2)

(U5) $(B{\leftarrow\!\diamond}A) \wedge C \rightarrow B{\leftarrow\!\diamond}(A \wedge C)$
v. handling of disjunctive input

(U2.2) If $B \rightarrow A$ then $B{\leftarrow\!\diamond}A \rightarrow B$
...

Reexamining the explanation of updates

1. belief base B = what is believed by an agent about the 'the actual world'
2. sensors information A about facts of the actual world
3. A describes an event in the actual world
4. agent changes B such that $B \leftarrow\!\diamond A$ as close as possible to what agent believes to know about the actual world
5. sensing can be noisy
6. agent ignores whether there is misperception or not

$WSS\downarrow$: a critique of conservativity (U2.2)

(U2.2) If $B \rightarrow A$ then $B \leftarrow\!\diamond A \rightarrow B$

- in $WSS\downarrow$
  $p \leftarrow\!\diamond (p \vee q) \leftrightarrow (p \vee q)$
- example:
  - A coin shows heads ($Heads$).
  - Somebody tosses the coin, but I can't see the outcome ($Heads \vee Tails$).
  - $Heads \leftarrow\!\diamond (Heads \vee Tails) \leftrightarrow (Heads \vee Tails)$

Integrity constraints – 1

alias domain constraints, domain laws, . . .

$$IC = \neg(Heads \wedge Tails)$$

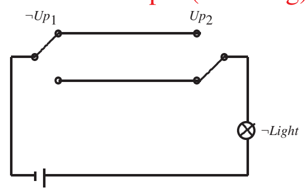
- wanted:
  $Heads \leftarrow\!\diamond_{IC} Tails \rightarrow \neg Heads$
- standard proposal:

$$B \leftarrow\!\diamond_{IC} A = B \leftarrow\!\diamond (A \wedge IC)$$

- $Heads \leftarrow\!\diamond (Tails \wedge \neg(Heads \wedge Tails)) \leftrightarrow Heads \wedge \neg Tails$

Integrity constraints – 2

- counterexample (Ginsberg):



  - $IC = (Up_1 \leftrightarrow Up_2) \leftrightarrow Light$
    $B = Up_1 \wedge Up_2 \wedge Light$
    $A = \neg Up_1$
  - intuitively: $B \leftarrow\!\diamond A \rightarrow Up_2$
  - . . . but not the case in PMA, MCD, $MCD*$, WSS, $WSS\downarrow$

alternative definitions of $B \leftarrow\!\diamond_{IC} A$?

WSS^{dep} = *WSS* plus dependence

(Herzig 1996)

- hypothesis: dependence function *dep*

$$dep(Up_1) = \{Up_1, Light\}$$

(given)

- $w \cdot A = \{v \in [A] | dist(w,v) \subseteq depA\}$
- $Up_2 \notin dep(Up_1)$:

$$\{Up_1, Up_2, Light\} \cdot Up_1 \subseteq [Up_2]$$

can be made syntax-independent by normal forming: $WSS{\downarrow}^{dep}$

similar proposal, but based on relation of causality:
$MPMA^{\gg}$ (Doherty et al., KR 1998)

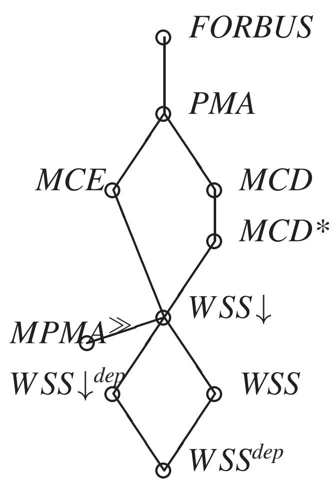
Overview: the desirable KM postulates

	(U1)	(U2.1)	(U3)	(U4)	(U8)
FORBUS	y	y	y	y	y
PMA	y	y	y	y	y
MCD	y	y	y	y	y
MCD*	y	y	y	y	y
WSS	y	y	y	n	y
$WSS{\downarrow}$	y	y	y	y	y
MCE	y	y	y	y	y
WSS^{dep}	y	y	y	n	y
$WSS{\downarrow}^{dep}$	y	y	y	y	y
$MPMA^{\gg}$	y	y	y	y	y

Overview: the undesirable KM postulates

	(U2.2)	(U5)	(U6)	(U7)
FORBUS	y	y	y	y
PMA	y	y	y	y
MCD	y	n	y	n
MCD*	y	n	y	n
WSS	n	n	n	y
$WSS{\downarrow}$	n	n	n	n
MCE	y	n	y	n
WSS^{dep}	n	n	n	y
$WSS{\downarrow}^{dep}$	n	n	n	n
$MPMA^{\gg}$	n	n	n	n

Overview: comparative strength



Overview: complexity

- PSPACE-complete:
  *MCD**
- Π_2^p-complete:
  PMA, *FORBUS*, *MCD*, *MCE*
- coNP-complete:
  WSS, $WSS\!\downarrow$, WSS^{dep}, $WSS\!\downarrow^{dep}$, $MPMA^{\gg}$

To conclude

- We started from selection function semantics ...
- ... refined minimal change:
  - closeness orderings
  - KM theory: faithful partial pre-orders of closeness
  - *FORBUS*, *PMA*: unique pre-order
- ... criticized KM theory:
  - handling of disjunctions
  - conservativity
  - handling of integrity constraints
- ... to finally get back to selection functions!
  - based on dependence

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