On the existence of general bar recursors

Tatsuji Kawai

Japan Advanced Institute of Science and Technology

This work is in the context of Heyting arithmetic in all finite types (HA^ω). In [1], Oliva and Steila introduced the following bar recursion scheme, called *general bar recursion*:

$$\xi(G, H)(a) = \begin{cases} G(a) & \text{if } S(a) \\ H(a, \lambda x.\xi(G, H)(a * \langle x \rangle)) & \text{otherwise} \end{cases}$$

where ξ is of type $(\tau^* \to \sigma) \to (\tau^* \to (\tau \to \sigma) \to \sigma) \to \tau^* \to \sigma$ and S is a monotone decidable bar of τ^* (τ is typically $\mathbb{N}$ or $\{0, 1\}$, but our argument works for any τ). Let us call ξ general bar recursor for S. Such a functional may not be shown to exist for every decidable bar. It is not difficult, however, to see that the decidable bar induction of type τ implies the existence of the bar recursor for any monotone decidable bar. The converse holds in HA^ω extended with the τ-branching tree type. In particular, we obtained the equivalents of the decidable fan theorem and the decidable bar induction (of the lowest type) in terms of existence of general bar recursors.

This is a joint work with Makoto Fujiwara (Waseda Institute of Advanced Study).

References

[1] P. Oliva and S. Steila. A direct proof of Schwichtenberg's bar recursion closure theorem. *J. Symbolic Logic*, pages 1–14, 2018.